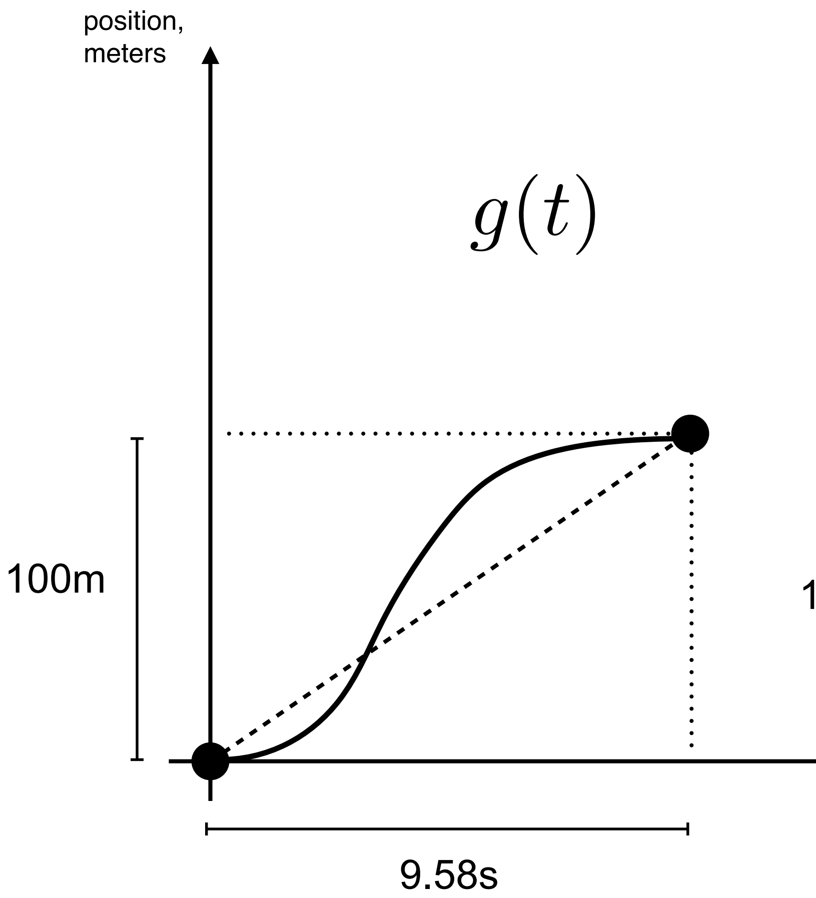
elementary math *refresher*

- (Mathematical) **functions** of one variable
- The **graph** of a function
- Adding or multiplying by a constant: how does the graph change?
- Straight lines, exponentials, logarithms,…

- First derivative of a function of one variable
- Definite and indefinite integrals (of a function of one variable)

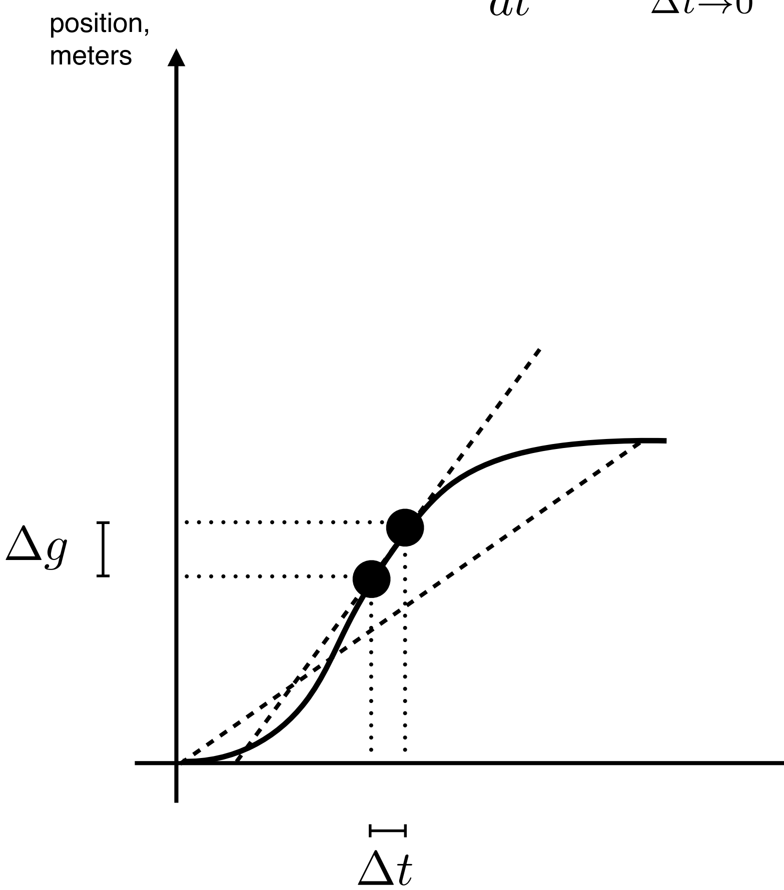


Usain BOLT

100m in 9.58s

10.43 m/s = 37.54 km/h

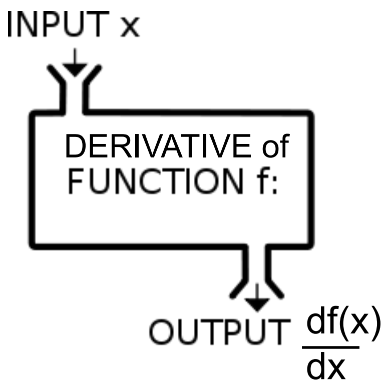
$$g'(t) = \frac{dg(t)}{dt} = \lim_{\Delta t \to 0} \frac{g(t+\Delta t) - g(t)}{\Delta t}$$



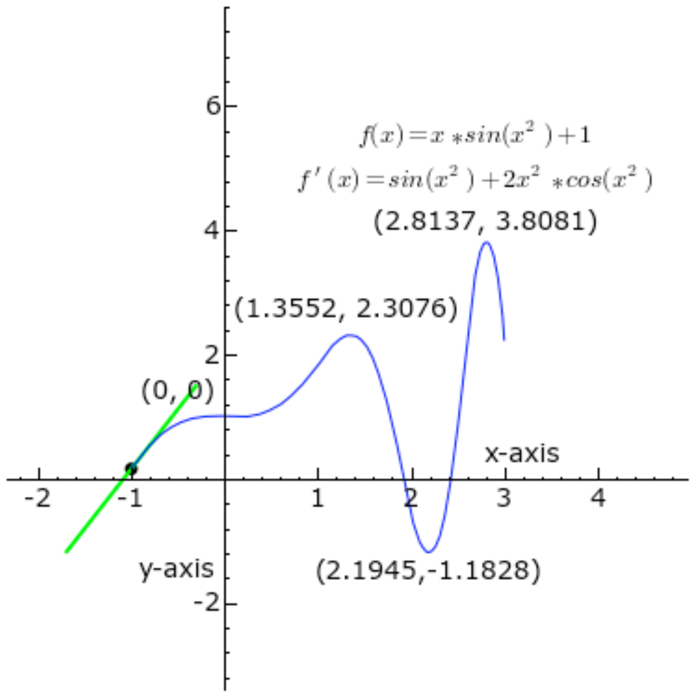
Usain BOLT

100m in 9.58s

The Derivative of a function

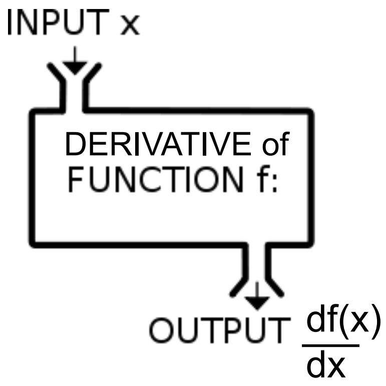


$$\frac{d}{dx}f(x) \quad f'(x) \quad \dot{f}(x)$$

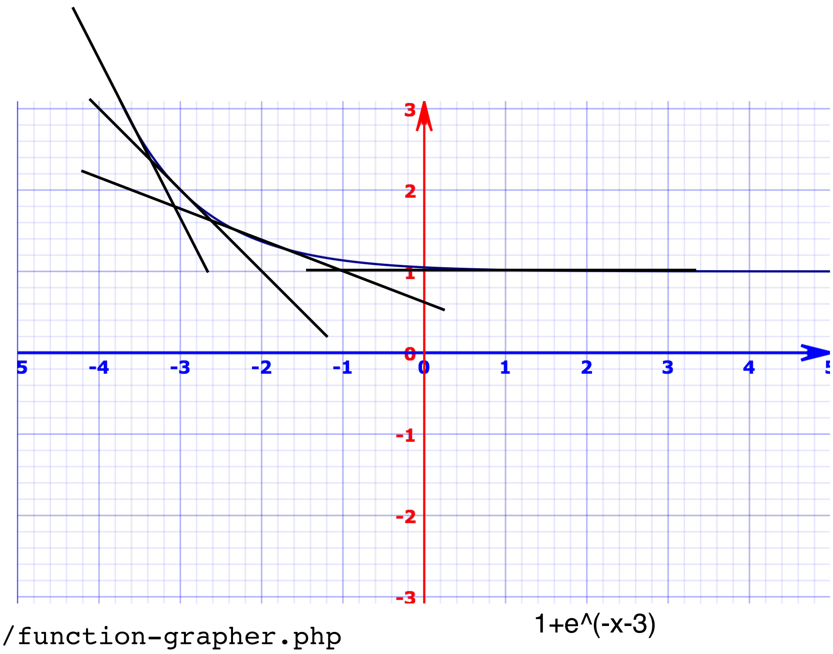


$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

The Derivative of a function



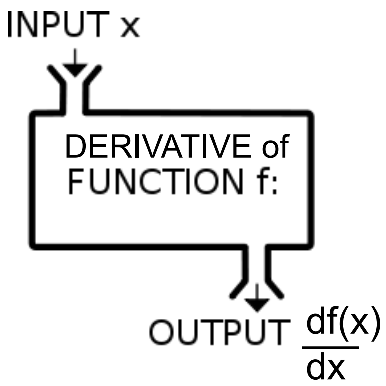
$$f(x) = 1 + e^{-x-3}$$



$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

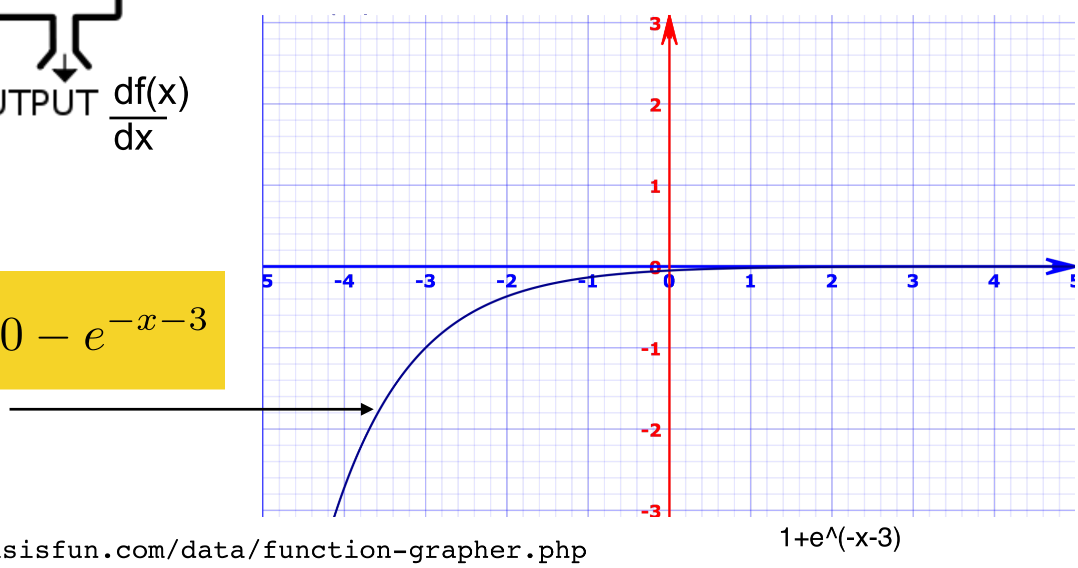
http://www.mathsisfun.com/data/function-grapher.php

The Derivative of a function



$$f(x) = 1 + e^{-x-3}$$

$$\frac{df(x)}{dx} = 0 - e^{-x-3}$$



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Derivatives of elementary functions

$$\frac{d}{dx}a = 0$$

$$\frac{d}{dx}x = 1 \qquad \frac{d}{dx}x^n = n\ x^{n-1}$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}sin(x) = cos(x) \qquad \frac{d}{dx}cos(x) = -sin(x)$$

$$\frac{d}{dx}\left[af(x)\right] = a\frac{d}{dx}f(x)$$

multiplication by a constant

$$\frac{d}{dx}\left[af(x) + bg(x)\right] = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x)$$

linear combination

$$\frac{d}{dx}\left[f(x)g(x)\right] = g(x)\frac{d}{dx}f(x) + f(x)\frac{d}{dx}g(x)$$

product

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{g(x)^2}$$

ratio

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{d}{du}f(u)\frac{d}{dx}g(x)$$

$$u = g(x)$$

composition (chain-rule)

$$\frac{d}{dx}[3sin(x)] = 3cos(x)$$

multiplication by a constant

$$\frac{d}{dx}[sin(x) - e^x] = cos(x) - e^x$$

linear combination

$$\frac{d}{dx}\left[x^4 ln(x)\right] = 4x^3 ln(x) + \frac{x^4}{x}$$

product

$$\frac{d}{dx}\left[\frac{1}{x}\right] = -\frac{1}{x^2}$$

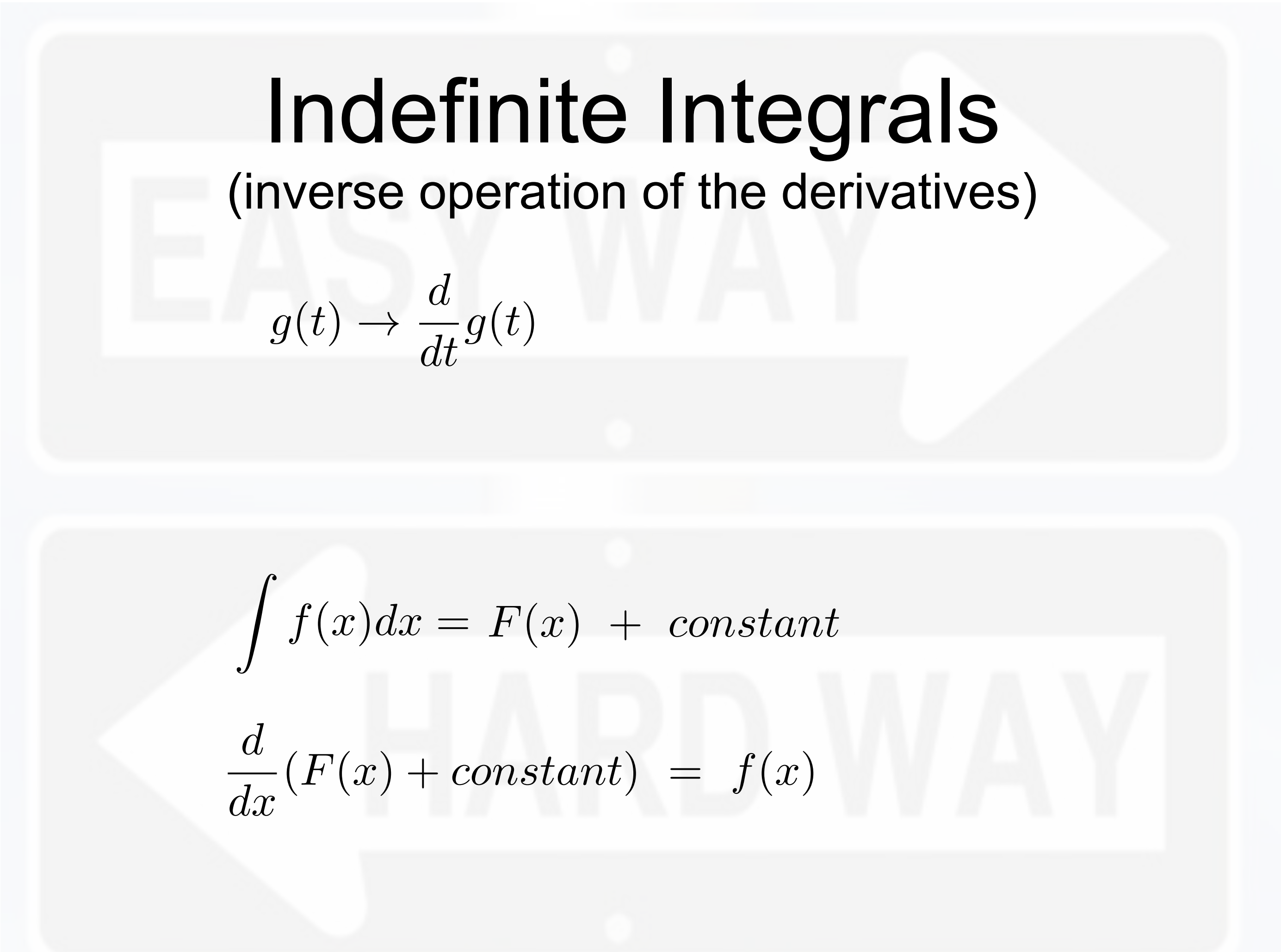
ratio

$$\frac{d}{dx}\left[e^{-x/a}\right] = -\frac{e^{-x/a}}{a}$$

composition (chain-rule)

Indefinite Integrals

(inverse operation of the derivatives)



$$g(t) \rightarrow \frac{d}{dt}g(t)$$

$$\int f(x)dx = F(x) \ + \ constant$$

$$\frac{d}{dx}(F(x) + constant) \ = \ f(x)$$

Integrals of elementary functions

$$\int a\ dx = a\ x + constant$$

$$\int x\ dx = \frac{1}{2}\ x^2 + constant \qquad \int x^n\ dx = \frac{1}{n+1}\ x^{n+1} + constant$$

$$\int \frac{1}{x}\ dx = ln(x) + constant$$

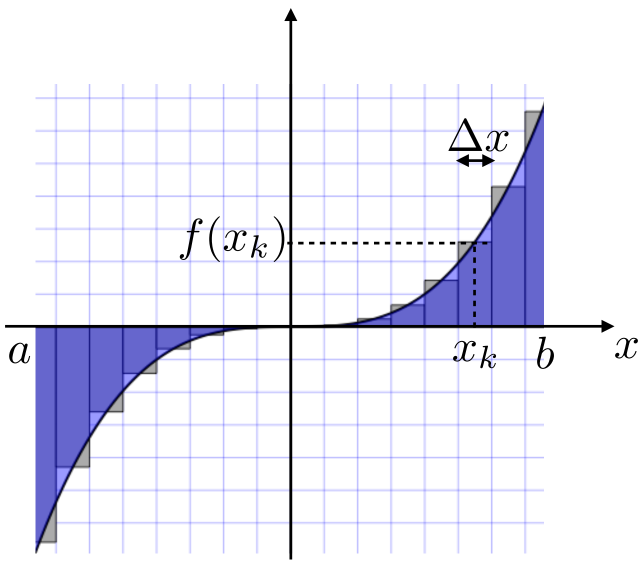
$$\int e^{a\ x}\ dx = \frac{1}{a} e^{a\ x} + constant$$

$$\int cos(x)\ dx = sin(x) + constant$$

$$\int sin(x)\ dx = -cos(x) + constant$$

Definite Integrals

(fundamental theorem of integral calculus)



$$A = f(a)\Delta x + ...$$
$$+ f(x_k)\Delta x + f(x_{k+1})\Delta x +$$
$$+ f(x_{k+2})\Delta x + f(x_{k+3})\Delta x + ...$$
$$... + f(b)\Delta x$$

$$A = \sum_{k=1}^{N} f(x_k)\Delta x$$

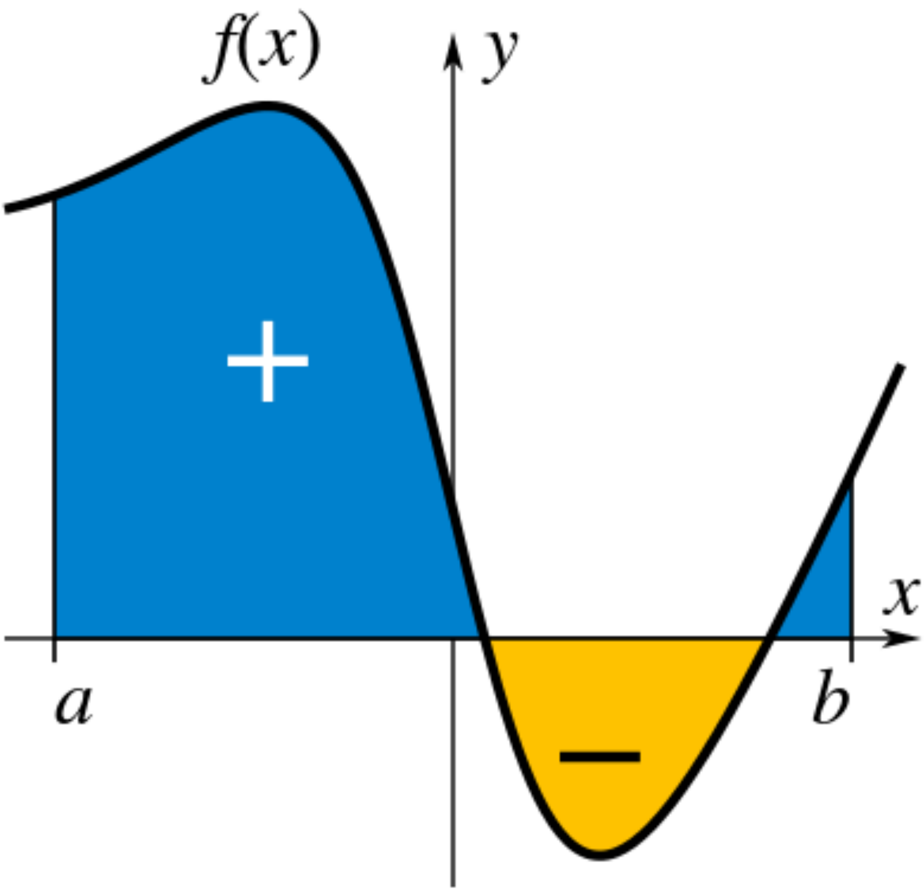
$$A = \sum_{k=0}^{N} f(x_k)\Delta x \to \int_a^b f(x)dx$$

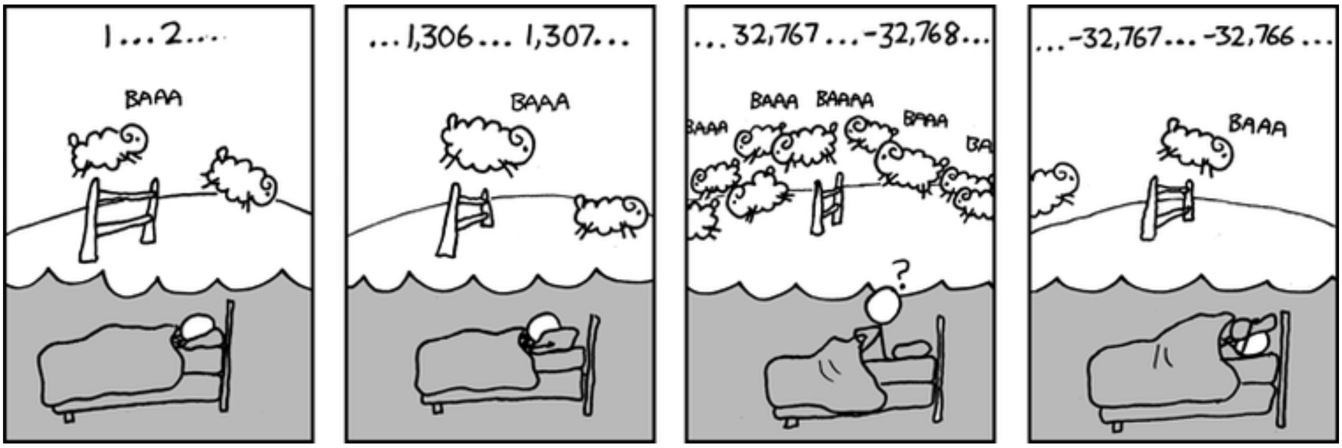
$$\int_a^b f(x)dx = F(b) - F(a)$$

$$\int_a^b \frac{d}{dx}F(x)dx = F(b) - F(a)$$

$$\int f(x)dx = F(x) \; + \; constant$$

$$\frac{d}{dx}(F(x) + constant) \; = \; f(x)$$





- https://www.khanacademy.org/math/algebra-home/alg-functions
- https://www.mathsisfun.com/calculus/introduction.html
- http://www-math.mit.edu/~djk/calculus_beginners/
- https://www.khanacademy.org/math/calculus-home